**Two-Sample T Tests**

* There are many situations in everyday life where we’d perform statistical analysis w/ 2 samples
* test a hypothesis about effect of 2 meds on curing an illness or the difference between the means of males and females on the SAT
* Assumptions for a hypothesis test on a *single* sample
* Random sample from the population
* Measure that sample statistic + make an inference about the population based on that sample.
* When we work with 2 *independent* samples, h(0) assumes that if samples are selected at random, the samples will vary only by chance + the difference will not be statistically significant.
* In short, *w/ independent samples we assume the scores of 1 sample do NOT affect the other.*
* 2 samples of data are **dependent** when each score in 1 sample is paired w/ a specific score in the other sample.
* Dependent samples can occur in 2 scenarios
* a group may be measured twice (pretest-posttest)
* an observation in 1 sample is matched w/ an observation in the 2nd sample
* Such as when researchers measure attitudes or behaviors of twins.
* The set up for the **independent sample t-test** is very similar to the single sample t-test.



* Notice our hypothesis statements have 2 population means, denoting the fact that we will be testing whether the means of two *separate* populations are equal to one another.
* An equivalent way of writing the hypotheses is as follows:



* Let’s see how the new hypothesis statements look in our t-statistic formula:



* x¯1 −x¯2 is the difference between the sample means
* µ1 −µ2 is the difference between the hypothesized population means
* SE(x¯1 − x¯2) = standard error *of the difference* between the sample means



* This standard error is called an **unpooled** **standard error**, b/c the standard deviation of *each* sample is considered.
* Finally, just like a single sample t-test, we’ll need degrees of freedom so we can find the correct critical value.
* Formula for degrees of freedom for an unpooled standard error independent samples t-test:



* *Technology automatically calculates this formula for us.*
* When we solve the unpooled independent samples t-test by hand, the conservative approach is to use the lowest n of the 2 groups minus one for dF
* Just like the single sample t-test, the independent samples t-test has some assumptions we must consider in order for the test to be valid:
* A random sample of each population is used.
* The random samples are each made up of independent observations
* Each sample is independent of one another
* The population distribution of each population must be nearly normal, or the sample size large
* Notice the “new” assumption 🡪 now w/ 2 means being examined, we need to make sure those 2 means are independent of one another.
* **Independence** = if an observation is assigned to 1 group, it *CANNOT* also be recorded in the other group.
* Oftentimes, independent groups are things like males vs. females, Astros vs. Rangers fans, tall vs. short, etc.
* Ex: The head of the English Dept. is interested in the difference in writing scores between freshman English students taught by different teachers.
* Incoming freshmen are randomly assigned to 1 of 2 teachers + given a standardized writing test after the 1st semester.
* We take a sample of 8 students from 1 class + 9 from the other.
* Is there a difference in achievement on the writing test between the two classes?



* 1) ID the hypotheses:
* We will be testing to see if the mean score of the 2classes are equal to one another:

H0 : µ1 = µ2

HA : µ1 != µ2

* 2) ID appropriate significance level + confirm the test assumptions.
* Use alpha = 0.05.
* We’re told students were randomly assigned + we assume that they did not switch classes (for independence), that student scores are independent from one another, + that the underlying population of students in each class is nearly normal in the distribution of scores
* 3) Analyze the data and generate the test statistic.
* t = -3.498 t-critical value for dF of 7 = +/- 2.365
* t is outside the t-critical value 🡪 we reject h(0) + conclude the populations of students in the 2 classes significantly differ in their standardized test scores at the end of the semester.
* As the 2 classes were randomly assigned, we can plausibly conclude the difference in the scores was due to the class assignment + whatever teaching technique was used
* As mentioned earlier, dependent sample are a bit different from the independent samples t-test.
* Another name for the dependent samples t-test is the **paired samples t-test**
* In some way, the 2 variables we will be testing will be paired/related to one another.
* Just b/c we used the word “related” *don’t think correlation*.
* This is still a t-test, and we’ll be testing questions about the means of variables.
* The **dependent samples t-test** is different from the independent samples in that:
* 1) In the *dependent* samples t-test, our hypothesis statement looks like:

H0 : δ = 0

HA : δ != 0

* Delta = the representation of “difference” 🡪 So the hypothesis of the *dependent* samples t-test is that the “difference” between 2 variables is 0.
* That sure sounds a lot like the hypothesis of an independent samples t-test when we test the difference between 2 population means.
* *The difference is subtle but important*.
* In an *independent* samples t-test we test the difference between 2 MEANS 🡪 calculate the mean for each group + then compare them.
* In a *dependent* samples t-test, we look at the difference between 2 VARIABLES w/in a *single observation*.
* Ex: We’re told students were tested at the end of the semester = just 1 time of testing.
* But what if all students were tested when they came into class (basic knowledge test) + then *again* at the end of the semester.
* Assuming the test was the same/very close both times the student saw it, then any change in the scores should represent knowledge gain over the course of the semester.
* B/c all students have 2 scores, the dependent samples t-test allows us to take the difference between the 2 scores for every student.
* *This difference score is only one column of data.*



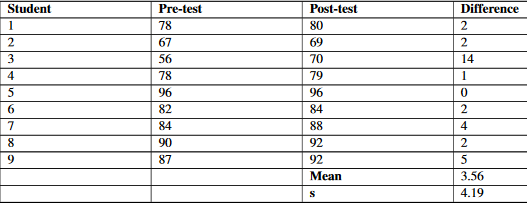
* d¯ = average of the difference between paired variables
* SEd¯ = standard error of the difference variable
* Notice d¯ in the numerator 🡪 the average of new variable of differences.
* Ex: If there was no knowledge change over the semester, the average of the differences in scores for each student = 0
* That’s why h(0) states that delta (average difference in scores for the entire population of students) = 0.
* The standard error in this formula is the standard error for *the variable representing the difference*, which is made up of the *standard deviation of the differences*.
* This standard deviation of the differences formula should look a lot like a simple standard deviation:



* And the formula for the standard error is:



* Our dF are similar to earlier, but this time since the test is only concerned about a *new* variable of differences for each subject, we can use the (n-1) dF formula, where n = pairs of data
* If there were 8 students in the class + each was measured twice, we’d still have 8 difference scores – 1 for each student.
* The assumptions of the dependent/paired samples t-test are very much like the single sample t-test assumptions (after all, the test is only concerned with a single variable (the difference))
* The sample of differences (hence the sample of paired observations) is random.
* The paired observations are independent of one another.
* The distribution of population differences must be nearly normal, or the size of the paired observation sample is large.
* Ex: Teacher wants to determine effectiveness of her statistics lesson + gives a simple skills test to 9 students before the start of class (pre-test) + the *same* skills test to the *same* students at the end of class (post-test).



* 1) Clearly state the Null and Alternative Hypothesis.
* The instructor is interested in improvement over the semester for each of her students.
* Since there are 2 measures for *each* student + those measures are paired, so we’ll need to use the dependent samples t-test.
* We assume the normal state of affairs would be no change due to chance (so delta = 0).
* H0 : δ = 0
* HA : δ 6= 0
* 2) ID the appropriate significance level + confirm the test assumptions.
* Assume alpha = 0.05 + students in class are random, independent of one another, + that the distribution of all possible difference scores in the population are nearly normally distributed
* 3) Analyze the data.
* We have the mean + SD for the data, *not* for each time variable (pre + post), but for the *difference* between the 2 variables for each student
* This column of differences is what we’ll use in the test.
* 4) Interpret your results.
* t-statistic is greater than t-critical value = reject h(0) = conclude there was in fact a change in student performance from the Pre-test to the Post-test.